Q. 67 The salaries of Ramesh, Ganesh and Rajesh were in the ratio 6:5:7 in 2010, and in the ratio 3:4:3 in 2015. If Ramesh's salary increased by 25% during 2010-2015, then the percentage increase in Rajesh's salary during this period is closest to

1. 8 2. 7 3. 9 4. 10

Q. 68 If x is a real number, then $\sqrt{\log_e \frac{4x - x^2}{3}}$ is a real number if and only if

1. $1 \le x \le 2$ 2. $-3 \le x \le 3$ 3. $1 \le x \le 3$ 4. $-1 \le x \le 3$

Q. 69 In an examination, Rama's score was one-twelfth of the sum of the scoresof Mohan and Anjali. After a review, the score of each of them increased by 6. Therevised scores of Anjali, Mohan, and Rama were in the ratio 11:10:3. Then Anjali'sscore exceeded Rama's score by

1. 24 2. 26 3. 32 4. 35

Q. 70: How many pairs (m,n) of positive integers satisfy the equation 2 2 m 105 n ?

Q. 71: Anil alone can do a job in 20 days while Sunil alone can do it in 40 days. Anil starts the job, and after 3 days, Sunil joins him. Again, after a few more days, Bimal joins them and they together finish the job. If Bimal has done 10% of the job, then in how many days was the job done?

1. 14 2. 13 3. 15 4. 12

Q. 72: Two circles, each of radius 4 cm, touch externally. Each of these two circles is touched externally by a third circle. If these three circles have a common tangent, then the radius of the third circle, in cm, is

1. 2 2. $\pi / 3$ 3. $1/2$ 4. 1

Q. 73: In an examination, the score of A was 10% less than that of B, the score ofB was 25% more than that of C, and the score of C was 20% less than that of D. IfA scored 72, then the score of D was

Q. 74: A cyclist leaves A at 10 am and reaches B at 11 am. Starting from 10:01 am, every minute a motor cycle leaves A and moves towards B. Forty-five such motor cycles reach B by 11 am. All motor cycles have the same speed. If the cyclist had doubled his speed, how many motor cycles would have reached B by the time the cyclist reached B?

1. 23 2. 20 3. 15 4. 22

Q. 75: The average of 30 integers is 5. Among these 30 integers, there are exactly 20 which do not exceed 5. What is the highest possible value of the average of these 20 integers?

1. 4 2. 3.5 3. 4.5 4. 5

Q. 76: John jogs on track A at 6 kmph and Mary jogs on track B at 7.5 kmph. The total length of tracks A and B is 325 metres. While John makes 9 rounds of track A, Mary makes 5 rounds of track B. In how many seconds will Mary make one round of track A?

Q. 77: In a six-digit number, the sixth, that is, the rightmost, digit is the sum of the first three digits, the fifth digit is the sum of first two digits, the third digit is equal to the first digit, the second digit is twice the first digit and the fourth digit is the sum of fifth and sixth digits. Then, the largest possible value of the fourth digit is

Q. 78: The quadratic equation $x^2 \quad bx \quad c \quad 0$ has two roots 4a and 3a, where a is an integer. Which of the following is a possible value of $b^2 + c$?

1. 3721 2. 549 3. 427 4. 361

Q. 79: The real root of the equation $2^{6x} + 2^{3x+2} - 21 = 0$ is

1. $\frac{\log_2 7}{3}$
2. $\log_2 9$
3. $\frac{\log_2 3}{3}$
4. $\log_2 27$

Q. 80: Let $a_1, a_2,$ ....be integers such that

$a_1 - a_2 + a_3 - a_4 + \cdots + (-1)^{n-1} a_n = n,$ for all $n \geq 1$

Then $a_{51} + a_{52} + \cdots + a_{1023}$ equals

1. -1 2. 10 3. 0 4. 1

Q. 81: Two ants A and B start from a point P on a circle at the same time, with A moving clock-wise and B moving anti-clockwise. They meet for the first time at 10:00 am when A has covered 60% of the track. If A returns to P at 10:12 am, then B returns to P at

1. 10 : 25am 2. 10 :18am 3. 10 : 27am 4. 10 : 45am

Q. 82: In a triangle ABC, medians AD and BE are perpendicular to each other, and have lengths 12 cm and 9 cm, respectively. Then, the area of triangle ABC, in sq cm, is

1. 68 2. 72 3. 78 4. 80

Q. 83: In 2010, a library contained a total of 11500 books in two categories - fiction and nonfiction. In 2015, the library contained a total of 12760 books in these two categories. During this period, there was 10% increase in the fiction category while there was 12% increase in the non-fiction category. How many fiction books were in the library in 2015?

1. 6000 2. 6160 3. 5500 4. 6600

Q. 84: The strength of a salt solution is p% if 100 ml of the solution contains p

grams of salt. Each of three vessels A, B, C contains 500 ml of salt solution of strengths 10%, 22%, and 32%, respectively. Now, 100 ml of the solution in vessel A is transferred to vessel B. Then, 100 ml of the solution in vessel B is transferred to vessel C. Finally, 100 ml of the solution in vessel C is transferred to vessel A.
The strength, in percentage, of the resulting solution in vessel A is

1. 12 2. 14 3. 13 4. 15

Q. 85: How many factors of 24 35 104 are perfect squares which are greater than 1 ?

Q. 86: What is the largest positive integer n such that $\frac{n^2+7n+12}{n^2-n-12}$ is also positive integer ?

1. 8 2. 12 3. 16 4. 6

Q. 87: Let a, b, x, y be real numbers such that $a^2+b^2=25, x^2+y^2=169$, and $ax+by+65$. If $k=ay\text{-}bx$, then

1. $k=0$
2. $0<k\leq\frac{5}{13}$
3. $k=\frac{5}{13}$
4. $k>\frac{5}{13}$

Q. 88: Let A be a real number. Then the roots of the equation $x^2-4x-\log_2 A=0$ are real and distinct if and only if

1. $A>1/16$ 2. $A>1/8$ 3. $A<1/16$ 4. $A<1/8$

Q. 89: A shopkeeper sells two tables, each procured at cost price p, to Amal and Asim at a profit of 20% and at a loss of 20%, respectively. Amal sells his table to Bimal at a profit of 30%, while Asim sells his table to Barun at a loss of 30%. If the amounts paid by Bimal and Barun are x and y, respectively, then (x −y) / p equals

1. 0.7 2. 1 3. 1.2 4. 0.50

Q. 90: Mukesh purchased 10 bicycles in 2017, all at the same price. He sold six of these at a profit of 25% and the remaining four at a loss of 25%. If he made a total profit of Rs. 2000, then his purchase price of a bicycle, in Rupees, was

1. 8000 2. 6000 3. 4000 4. 2000

Q. 91: John gets Rs 57 per hour of regular work and Rs 114 per hour of overtime work. He works altogether 172 hours and his income from overtime hours is 15% of his income from regular hours. Then, for how many hours did he work overtime?

Q. 92: A man makes complete use of 405 cc of iron, 783 cc of aluminium, and 351 cc of copper to make a number of solid right circular cylinders of each type of metal.

These cylinders have the same volume and each of these has radius 3 cm. If the total number of cylinders is to be kept at a minimum, then the total s urface area of all these cylinders, in sq cm, is

1. 8464π 2. 928π 3. $1044(4+\pi)$ 4. $1026(1+\pi)$

Q. 93: Let ABC be a right-angled triangle with hypotenuse BC of length 20 cm. If AP is perpendicular on BC, then the maximum possible length of AP, in cm, is

1. 10 2. $6\sqrt{2}$ 3. $8\sqrt{2}$ 4. 5

Q. 94: The base of a regular pyramid is a square and each of the other four sides is an equilateral triangle, length of each side being 20 cm. The vertical height of the pyramid, in cm, is

1. $8\sqrt{3}$ 2. 12 3. $5\sqrt{5}$ 4. $10\sqrt{2}$ Q.

95: Let f be a function such that f (mn) = f (m) f (n) for every positive integers m and n. If f (1), f (2) and f (3) are positive integers, f (1) < f (2), and f (24) = 54, then f (18) equals

Q. 96: If $(2n+1)+(2n+3)+(2n+5)+\ldots+(2n+47) = 5280$, then what is the value of $1+2+3+\ldots+n$?

Q. 97: If $5^x - 3^y = 13438$ and $5^{x-1} - 3^{y+1} = 9686$ then x+y equals

Q. 98: Let A and B be two regular polygons having a and b sides, respectively. If b = 2a and each interior angle of B is 3/2 times each interior angle of A, then each interior angle, in degrees, of a regular polygon with a + b sides is

Q. 99: The number of common terms in the two sequences: 15, 19, 23, 27, . . . 415 and 14, 19, 24, 29, . . . , 464 is

1. 18 2. 19 3. 21 4. 20

Q. 100: Amal invests Rs 12000 at 8% interest, compounded annually, and Rs 10000 at 6% interest, compounded semi-annually, both investments being for one year. Bimal invests his money at 7.5% simple interest for one year. If Amal and Bimal get the same amount of interest, then the amount, in Rupees, invested by Bimal is

Solution 67 _ to _ 100

Solution 67:

Let their salaries in 2010 be 6x, 5x and 7x respectively.
Also, let their salaries in 2015 be 3y, 4,y and 3y respectively
Given, $3y = 1.25 \times 6x$
Ory = 2.5 x.
Therefore, salary of Rajesh in 2015 = 3y = 3 × 2.5x = 7.5 x

Percentage increase = $\left(\frac{7.5x-7x}{7x}\right)\times 100 \approx 7\%$

Solution 68:

The expression will be real only if

$$\log_e \frac{4x-x^2}{3} \geq 0$$

Or $\frac{4x-x^2}{3} \geq e^0$

$$\Rightarrow \frac{4x-x^2}{3} \geq 1$$

$$\Rightarrow 4x-x^2 \geq 3$$

$$\Rightarrow x^2-4x+3 \leq 0$$

$$\Rightarrow (x-1)(x-3) \leq 0$$

$$1 \leq x \leq 3$$

Solution 69:

Let their scores after review be 11x, 10x, and 3x respectively.
Therefore, their scores before review was: (11x-6), 10x-6) and (3x-6) respectively.
Given, Rama's score was one-twelfth of the sum of the scores of Mohan and Anjali.

$$\Rightarrow (3x-6) = \frac{1}{12}\left[(11x-6)+(10x-6)\right]$$

$$\Rightarrow 12(3x-6) = 21x-12$$

$$\Rightarrow 36x-72 = 21x-12$$

$$\Rightarrow 36x-21x = 72-12 = 60$$

$$\Rightarrow x = 4$$

Now, Anjali’s score – Rama’s score = (11x-6)-(3x-6)=8x =32.

Solution 70:

Shortcut:

Number of pairs = $\frac{number\ \ of\ \ \ facors\ \ 105}{2}$

105 = 3 × 5 × 7
Number of factors = 2 × 2 × 2 = 8
Hence, required number of pairs =8/2 =4
Detailed Explanation:

$m^2 + 105 = n^2$ c

$\Rightarrow n^2 - m^2 = 105$

$\Rightarrow (n-m)(n+m) = 105$

Since m and n are positive integers, $(n-m)<(n-m)$

Splitting 105 in two factors, we get

$$\Rightarrow (n-m)(n+m)=1\times 105$$

For $(n-m)=1$ and $(n+m)=105$, $(m,n)=(52,53)$

$$\Rightarrow (n-m)(n+m)=3\times 35$$

For $(n-m)=3$ and $(n+m)=35$, $(m,n)=(16,19)$

$$\Rightarrow (n-m)(n+m)=5\times 21$$

For $(n-m)=5$ and $(n+m)=21$, $(m,n)=(8,13)$

$$\Rightarrow (n-m)(n+m)=7\times 21$$

For $(n-m)=7$ and $(n+m)=21$, $(m,n)=(4,11)$

Hence there are four pairs.

Solution 71:

Let the work be of 40 units

Amount of work done by Anil in one day = 40/20=2 units

Amount of work done by Sunil in one day = 20/20=1 units

Bimal does 10% work i.e. 4 units.

Rest 40-4=36 units is done by Anil and Sunil.

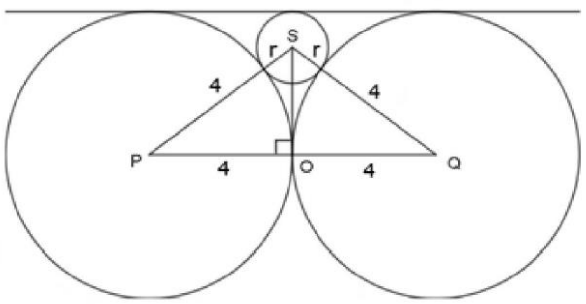
Let Anil took x days. Therefore, Sunil took (x-3) days. Therefore,

$2\times x+1\times(x-3)=36$

Or x = 13 days.

Solution 72:

Refer to the figure



SO=4-r.

Applying Pythagoras theorem in triangle POS, we get

$$(4+r)^2 = 4^2 + (4-r)^2$$

$$\Rightarrow (4+r)^2 - (4-r)^2 = 16$$

$$\Rightarrow 4 \times 4 \times r = 16$$

$$\Rightarrow r = 1$$

Solution 73:

Given A= 72

Also, A=0.9×B => B=A/0.9=72/0.9=80.

And B=1.25×C => C = B/1.25=80/1.25=64

And C=0.8×D => D =C/0.8 = 64/0.8=80.

Solution 74:

Time taken by cyclist to cover the distance AB = 60 min

Given, starting from 10:01 am, every minute a motor cycle leaves A and moves towards B.

Forty-five such motor cycles reach B by 11 am.

Also, the speed of all the motor cycles is same.

That means that the 45th moto cyclevwhich started at 10:45 am, reached B exactly at 11

am. Rest all reached B some time before B.

Therefore, each motor cycle takes 15 min to cover the distance AB.

Now, if the cyclist doubles his speed, then he will reach B in 30 min i.e. at 10:30 am.

So, the 15th motor cycle (started at 10:15 am from A) would be the last motor cycle to reach point B at 10:30 am.

Hence, there will be 15 motor cycles would have reached B by the time the cyclist reached B.

Solution 75:

Let a be the average of 20 numbers whose average does not exceed 5.

Let b be the average of rest of the 10 numbers. Clearly, b>5 i.e. the average of these numbers exceeds 5.

Therefore,

$$30 \times 5 = 20a + 10b$$

$$\Rightarrow 2a + b = 15$$

$$\Rightarrow b = 15 - 2a$$

Going by the options, we can say that when a=4.5, b=6 which satisfies all the conditions.

Solution 76:

Speed of John = 6kmph = $6 \times \frac{5}{18} = \frac{5}{3} m/s$

Speed of mary = 7.5 kmph = $7.5 \times \frac{5}{18} = \frac{25}{12}$ m/s

Let the track length of A and B be x and y respectively.
Given, x + y = 325 .... (1)

Time taken by John to cover one round of $A = \frac{x}{5/3}$ sec

Therefore, time taken to cover 9 rounds = $9 \times \frac{x}{5/3} = \frac{27}{5} x$ sec

Therefore, time taken to cover 5 rounds = $5 \times \frac{y}{25/12} = \frac{12}{5} y$ sec

As per the condition :

$$\frac{27}{5} x = \frac{12}{5} y$$

$$\Rightarrow \frac{x}{y} = \frac{12}{27} = \frac{4}{9}$$

Putting in equation (1) we get x=100 and y =225.

Time taken by Mary to cover one round of A = $\frac{100}{25/12} = 48$ sec

Solution 77:

Let the number be ABCDEF, where A, B, C, D, E, and F be the digits.
Given,
C=A
B=2A
F=A+B+C=A+2A+A=4A
E=A+B=A+2A=3A
D=E+F=3A + 4A=7A.
Since A and D both are digit, the maximum possible value of A=1. Therefore, the maximum value of D=7.

Solution 78:

Sum of roots = 4a+3a=7a=-b
Or b=-7a
Product of roots = 4a×3a =c
Or $c = 12a^2$
Now, $b^2 + c = (-7a)^2 + 12a^2 = 61a^2$
Comparing the options.
Option 1: $61a^2 = 3721 \Rightarrow a^2 = 61$, clearly a is not an integer.
Option 2: $61a^2 = 549 \Rightarrow a^2 = 9$ we can have a =-3 or 3 (an integer)
Option 3: $61a^2 = 427 \Rightarrow a^2 = 7$, clearly a is not an integer.
Option 4 : $61a^2 = 361 \Rightarrow a^2 = \frac{361}{61}$ clearly a is not an integer.

Solution 79:

$$2^{6x}+2^{3x}\times 2^{2}-21=0$$

Take $2^{3x}=y$

$$\Rightarrow y^{2}+4y-21=0$$

$$\Rightarrow (y-3)(y+7)=0$$

$$\Rightarrow y=3 \text{ or } y=-7$$

$$\Rightarrow 2^{3x}=3 \text{ or } 2^{3x}=-7\{\text{ No solution }\}$$

$$\Rightarrow 3x=\log_{2}3$$

$$\Rightarrow x=\frac{log_{2}3}{3}$$

Solution 80:

for $n=1,\quad a_1=n\Rightarrow a_1=1$

for $n=2,\quad a_1-a_2=2\Rightarrow a_2=-1$

for $n=3,\quad a_1-a_2+a_3=3\Rightarrow a_3=1$

for $n=4,\quad a_1-a_2+a_3-a_4=4\Rightarrow a_4=-1$

From the pattern, each odd term = 1 and each even term = -1

$$\Rightarrow a_{51}+a_{52}+\cdots+a_{1022}=0$$

Therefore the value is equal to $a_{1023}=1$

Solution 81:

Let the track length be 10x.

When they meet at 10 am, ant A travelled 6x of the distance and ant B travelled 4x of the distance.

Therefore, $\frac{Speed\ of\ ant\ A}{Speed\ of\ ant\ A}=\frac{6x}{4x}=\frac{3}{2}$

And, the ratio of time taken by A and B to cover the same distance = $\frac{2}{3}$

The distance by ant A from meeting point to point P was 4x. Similarly, the distance covered
by ant B from meeting point to point P was 6x.

Given, ant A took 12 min to reach P.

Therefore, to cover a distance of 4x, time taken by ant B = $\frac{3}{2}\times 12 = 18$ min.

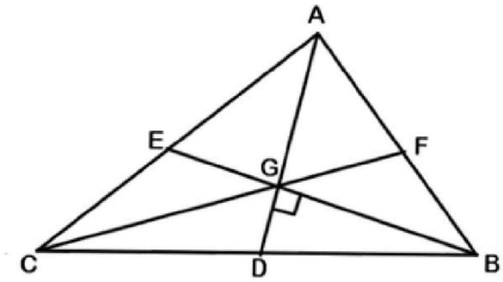
But, ant B has to cover a total of 6x distance.

Hence, the time required = $\frac{6x}{4x}\times 18 = 27$ min.

Therefore, ant B reaches P at 10:27 am.

Solution 82:

Refer to the figure below:



Draw the third median CF. We know the following facts:
1. The intersection point of medians i.e. centroid (G) divides each median into 2:1.
2. All three medians divide the triangle into 6 parts of equal area.

$$GD = \frac{1}{3}\times AD = \frac{1}{3}\times 12 = 4$$

$$GB = \frac{2}{3}\times BE = \frac{2}{3}\times 9 = 6$$

Area of triangle BGD = $\frac{1}{2}\times GB\times GD = \frac{1}{2}\times 6\times 4 = 12$

Hence, area of triangle ABC = $6\times 12 = 72$

Solution 83:

Let there number of fiction and non fiction books in 2010 be x and y respectively.

From the first condition:

$x + y = 11500....(1)$

From the second condition:

$1.1\times x + 1.2\times y = 11500\cdots(2)$

Solving both the equations, we get x = 6000.

In 2015, the number of fiction books = 1.1x = 6600

Solution 84:

Initial amount of salt in vessel A=10 gms per 100 ml, therefore in 500 ml the amount of salt =50 gms

Initial amount of salt in vessel B=22 gms per 100 ml, therefore in 500 ml the amount of salt =110 gms

Initial amount of salt in vessel C=32 gms per 100 ml, therefore in 500 ml the amount of salt =160 gms
When 100 ml is trasfered from A to B, the amount of salt now in B = 10+110=120 gms in 600ml.
The new concentration of salt in B = 120/600=20 gms per 100 ml.
Also, the amount of salt lef in A =50-10 =40 gms in 400ml.
Now, when 100 ml is trasfered from B to C, the amount of salt now in C = 20+160=180 gms in 600ml.
The new concentration of salt in C= 180/600=30 gms per 100 ml.
Finally, when 100 ml is trasfered from C to A, the amount of salt now in A = 30+40=70 gms in 500ml.
Therefore, the strength of salt in $A = \frac{70}{500} \times 100 = 14\%$

Solution 85:

$$2^4 \times 3^5 \times 10^4 = 2^8 \times 3^5 \times 5^4$$

For perfect squares, we have to take only even powers of the prime factors of the number.
The number of ways 2's can be used is 5 i.e. 2^0 , 2^2 , 2^4 , 2^6 , 2^8
The number of ways 3's can be used is 3 i.e. 3^0 , 3^2 , 3^4
The number of ways 5's can be used is 3 i.e. 5^0 , 5^2 , 5^4
Therefore, the total number of factors which are perfect squares = $5 \times 3 \times 3 \times 45$
But this also includes the number 1. Hence excluding 1, the required number is 45-1=44.

Solution 86:

$$\frac{n^2+7n+12}{n^2-n-12} = \frac{(n+3)(n+4)}{(n-4)(n+3)} = \frac{(n+4)}{(n-4)}$$

$$\Rightarrow \frac{(n+4)}{(n-4)} = \frac{(n-4+8)}{(n-4)} = 1 + \frac{8}{(n-4)}$$

The expression is positive integer if $\frac{8}{(n-4)}$ is integer.

Or (n-4) must be factor of 8.
For n to be largest, n-4=8
Or n =12

Solution 87:

Shortcut:
We can take a=5, b=0, x=13 and y=0 as values which satisfies all three equations.
Hence, $k = ay - bx = 5 \times 0 - 0 \times 13 = 0$

Solution 88:

For quadratic equation $ax^2+bx+c=0$ the, roots are real and distinct if $b^2-4ac>0$

Given, $x^2-4x-\log_2 A=0$

$$\therefore (-4)^2-4\times1\times(-\log_2 A)>0$$

$$\Rightarrow 16+4\log_2 A>0$$

$$\Rightarrow \log_2 A>-4$$

$$\Rightarrow A>2^{-4}$$

$$\Rightarrow A>\frac{1}{16}$$

Solution 89:

Cost of table for Aman = 1.2p

Cost of table for Asim = 0.8p

Aman sells to Bimal at 1.3×1.2p=1.56p =cost of table for Bimal = x

Asim sells table to Barun at 0.7×0.8p = 0.56p = cost of table for Barun = y

Therefore, $\frac{x-y}{p}=\frac{1.56p-0.56p}{p}=1$

Solution 90:

Let he cost of each bicycle be x.

From the given condition:

$$10x + 2000 = 6\times1.25x + 4\times0.75x$$

$$\Rightarrow x=4000$$

Solution 91:

Let the number of hours for regular and overtime work be x and y respectively.

We have two equations:

$$x + y = 172 \ldots(1)$$

$$114y = \frac{15}{100}\times57x \ldots(2)$$

On solving both the equations, we get x = 160 and y 12.

Hence, his overtime work =12 hours

Solution 92:

To get the minimum number of cylinders, the volume of each of the cylinder must be HCF of 405,783, and 351

$\Rightarrow HCF(405,783,351)=27$

Therefore, number of cylinders of iron $=\frac{405}{27}=15$

and, number of cylinders of aluminum $=\frac{783}{27}=29$

and, number of cylinders of copper $=\frac{351}{27}=13$

Hence, the total number of a cylinders $=15+29+13=57$

Also, volume of each cylinder =27 cc

$\Rightarrow \pi r^2 h=27$

$\Rightarrow \quad \pi\times 3^2\times h=27$

$\Rightarrow \quad h=\frac{3}{\pi}$

And total surface area of each cylinder $=2\pi r(r+h)$

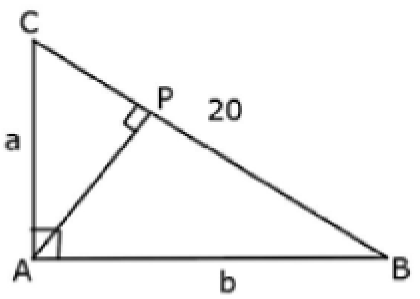
$=2\pi\times 3\left(3+\frac{3}{\pi}\right)=18(\pi+1)$

Hence, total surface area of 57 cylinders $=57\times 18(\pi+1)$

$=1026(\pi+1)$

Solution 93:

Refer to the figure:



For this right angle triangle, we have the following relations.

$a^2+b^2=20^2=400....(1)$ and

$AP=\frac{ab}{20}....(2)$

For maximum value of AP, we have to maximize the product ab.

Applying $AM\geq GM$ inequality we get

$$\frac{a^2+b^2}{2} \geq \sqrt{a^2 \times b^2}$$

$$\Rightarrow \frac{400}{2} \geq ab$$

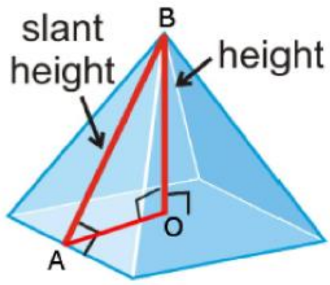
$$\Rightarrow ab \leq 200$$

Hence the maximum value of ab =200.

Therefore, the maximum value of AP $= \frac{200}{20} = 10$

Solution 94:

From the diagram, it is obvious that AB is the height of the equilateral triangle and is also the slant height of the pyramid.



Therefore, $AB = \frac{\sqrt{3}}{2} \times side = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$

And $AO = \frac{1}{2} \times side = \frac{1}{2} \times 20 = 10$

Applying Pythagoras theorem in triangle AOB

$$OB^2 = AB^2 - OA^2$$

$$= \left(10\sqrt{3}\right)^2 - 10^2$$

$$= 200$$

Hence, the height of the pyramid (OB) $= 10\sqrt{2}$

Solution 95:

Given, f(mn)=f(m)f(m)

Also, f(24)=54

$\Rightarrow$ f(24) =2×3×3×3

$\Rightarrow$f(2×12)=f(2)f(12)=f(2)f(2×6)=f(2)f(2)f(6)=f(2)f(2)f(2×3)= f(2)f(2)f(2)f(3)=2×3×3×3

Given that f(1), f(2) and f(3) are all positive integers, by comparison, we get f(2) = 3 and f(3)=2. And we can safely take f(1)=1

Now, f(18)=f(2)(9)=f(2)f(3×3)=f(2)f(3)f(3)=3×2×2=12

Solution 96:

The sequence $(2n+1)+(2n+3)+(2n+5)+\ldots+(2n+47)=5280,$ is in arithmetic progression with first term (a) = 2n+1, common difference (d) = 2 and last term (t_n)=2n+47.

Let 'm' be the number of terms in this sequence.
The last term of A.P. is given by a+(n-1)d

$$\Rightarrow (2n+1)+(m-1)(2)=2n+47$$

$$\Rightarrow m=24$$

Also,

$$(2n+1)+(2n+3)+(2n+5)+\ldots+(2n+47)=5280,$$

$$=\frac{24}{2}\left[2(2n+1)+(24-1)\times 2\right]$$

$$=24(2n+1+23)=48(n+12)$$

Therefore, $48(n+12)=5280 \Rightarrow n=98$

Hence, $1+2+3+\ldots+n=\frac{n(n+1)}{2}=\frac{98\times 99}{2}=4851$

Solution 97:

Taking 2^{nd} equation
Taking 2^{nd} equation

$5^{x-1}+3^{y+1}=9686$, the last digit of 5^{x-1} will always be 5 for all positive integral values of x

The power cycle of 3 is:

$3^{4k+1} \equiv 3$

$3^{4k+2} \equiv 9$

$3^{4k+3} \equiv 7$

$3^{4k} \equiv 1$

Clearly 3^{y+1} must be in the form of 3^{4k} as the unit digit of R.H.S. =6

We have $3^4 = 81, \text{and } 3^8 = 6561$

Also, $9686 - 81 = 9605$ and $9686 - 6561 = 3125$

Observe that $3125 = 5^5$

Hence $5^{x-1} = 5^5$

or $x = 6$ and $3^{y+1} = 3^8 \Rightarrow y = 7$

(x=6 and y=7 also satisfies the first equation)

Therefore, $x + y = 6 + 7 = 13$

Solution 98:

The formula for each interior angle = $180 - \dfrac{360}{n}$ where 'n' is the side of the regular polygon

$$\Rightarrow 180 - \frac{360}{2a} = \frac{3}{2}\left(180 - \frac{360}{a}\right)$$

$$\Rightarrow 360 - \frac{360}{a} = 540 - \frac{3 \times 360}{a}$$

$$\Rightarrow \frac{2 \times 360}{a} = 180$$

$$\Rightarrow a = \frac{2 \times 360}{180}$$

$$\Rightarrow a = 4 \text{ and } b = 2a = 8$$

Polygon with each side $= a + b = 4 + 8 = 12$, will have each interior angle $= 180 - \dfrac{360}{12}$

=150

Solution 99:

Both the sequences are in arithmetic progression.

The common difference (d_1) for the first sequence = 4

The common difference (d_2) for the first sequence = 5

The first term common is 19.

The common terms will also be in arithmetic progression with common difference

$$LCM(d_1, d_2) = LCM(4,5) = 20$$

Let there be 'n' terms in this sequence, then the last term would be ≤ 415

i.e. $a + (n-1)d \leq 415$

$$\Rightarrow 19 + (n-1) \times 20 \leq 415$$

$$\Rightarrow (n-1) \times 20 \leq 415 - 19$$

$$\Rightarrow (n-1) \times 20 \leq 396$$

$$\Rightarrow (n-1) = \left[\frac{396}{20}\right]$$ where [ ] is the greatest integer

$$\Rightarrow (n-1) = 19$$

$$\Rightarrow n = 20$$

Solution 100:
Let the amount invested by Bimal be Rs. P
Given, the interest incomes for both are equal. Therefore,

$$\left[12000\left(1+\frac{8}{100}\right) - 12000\right] + \left[10000\left(1+\frac{3}{100}\right)^2 - 10000\right] = \frac{P \times 7.5 \times 1}{100}$$

Solving for P we get P = Rs. 20920